COLLECTIVE DYNAMICS, SELF-ORDERING AND LONG RANGE INTERACTIONS OF POINT-PARTICLES IN NANO-PHOTONIC OPTICAL FIELDS

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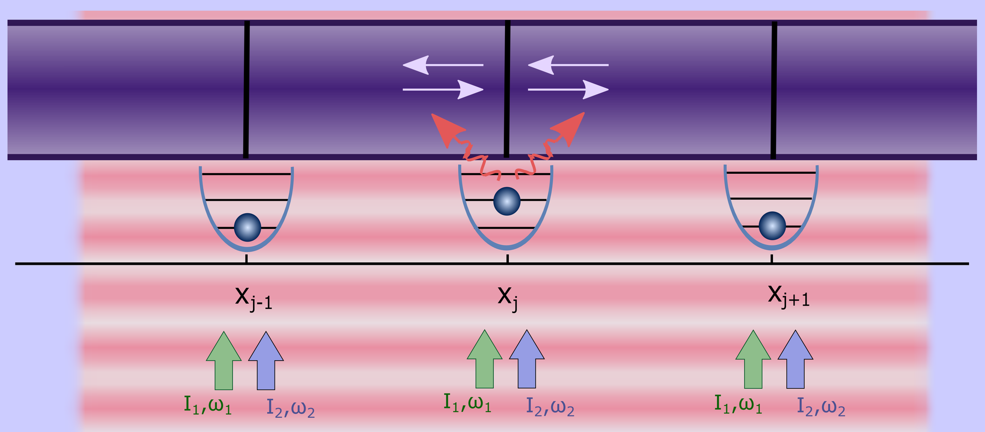
1 Introduction

Light-induced interactions between particles in the evanescent field of a nanofibre are periodic and of infinite range.
In 2009/2010 Rauschenbeutel and coworkers managed to trap laser-cooled neutral caesium atoms, interacting with the evanescent field surrounding an optical nano-fibre.
We try to develop a system to control the shape of the optical interaction potential by spectral design of the incoming illumination.
If each particle is only weakly coupled to the confined guided modes, the forces acting within a particle ensemble can be decomposed to pairwise interactions.
These two-particle interactions can be tailored as they are related to Fourier transforms with coefficients controlled by the intensities and frequencies of the illuminating lasers.
Implementing in a chain of trapped quantum particles as a versatile quantum simulator with almost arbitrary all-to-all interaction control.

2 Model



- N point-like polarizable particles each trapped in a harmonic potential along the evanescent field of an optical nano-fiber
- Illuminated transversely by different pump beams with different frequencies ω_i and intensities $I(k)$
- Each particle scatters light from transverse pump laser into fiber modes and between incoming and outgoing fields
- Collective scattering into propagating fiber modes induces long range interactions
- Neglect interference effects between distant spectral components
- Internal states are eliminated, consider only motional degrees of freedom

3 Weak scattering limit

- Assumption: strong pump field leads to large scattering rates into fibre compared to the reflection by each particle → neglect reflection
- Force on each particle i is the sum of all pair forces $F_{pair}(x_i,x_j)$

$$F_{i,N}=\sum_j\sum_k F_{pair}(x_i,x_j)=\sum_j\sum_k\frac{\sigma_{sc}I(k)\cos(k|x_i-x_j|)}{c}$$

- Position of the particles x_i, x_j, scattering cross section of the particle and the beam σ_{sc}, intensity scattered into the fibre by the particles $I(k)$, wavenumber k and speed of light c

4 Define a potential

- Two particle potential

$$-\partial_{x_j}U_{pair}(k,x_i,x_j)=F_{pair}(k,x_i,x_j)$$

- Potential for one particle i as a function of the position of the other particles

$$U_{j,N}(x_1,\dots,x_N)=\sum_{\substack{i=1\\i\neq j}}^{N}\sum_k\frac{\sigma_{sc}I(k)}{ck}\sin\big(k|x_i-x_j|\big)$$

- Potential of the system

$$U_{tot}(x_1,\dots,x_N)=\frac{1}{2}\sum_{j=1}^{N}\sum_{\substack{i=1\\i\neq j}}^{N}\sum_k\frac{\sigma_{sc}I(k)}{ck}\sin\big(k|x_i-x_j|\big)$$

- Note: Changing the position of one particle affects the potential of each other particle

5 Quantization of spatial degrees of freedom

- Introduce the harmonic oscillator potential with mass m, trap frequency ω_T and displacement of the particles Δ_i

$$U_{HO}=\sum_{i=1}^{N}\frac{m\omega_T}{2}\Delta_i^2$$

- Linearize around position of the harmonic oscillator $x_i\to x_{i,0}+\Delta_i$, $x_j\to x_{j,0}+\Delta_j$, with $\Delta_i,\Delta_j\ll 1$, and $d_{ij}=|x_{j,0}-x_{i,0}|$

$$H=\frac{\sigma_{sc}}{2c}\sum_k I(k)\sum_{j=1}^{N}\left(2\sum_{i=1}^{j-1}\left(\frac{1}{k}\sin\big(kd_{ij}\big)+\big(\Delta_j-\Delta_i\big)\cos\big(kd_{ij}\big)\right)-\sum_{i=1}^{N}\left(\frac{k\big(\Delta_j-\Delta_i\big)^2}{2}\sin\big(kd_{ij}\big)\right)\right)+\sum_{i=1}^{N}\left(\frac{P_i^2}{2m}+\frac{m\omega_T^2}{2}\Delta_i^2\right)$$

- Quantize motion of the particles

$$\Delta_i=\sqrt{\frac{\hbar}{2m\omega_T}}(a_i+a_i^\dagger)=\alpha(a_i+a_i^\dagger),\qquad P_i=i\sqrt{\frac{\hbar m\omega_T}{2}}(a_i^\dagger-a_i).$$

$$H=\frac{\sigma_{sc}\alpha}{c}\sum_k I(k)\sum_{j=1}^{N}\left(\sum_{i=1}^{j-1}\cos(kd_{ij})\underbrace{(a_j+a_j^\dagger-a_i-a_i^\dagger)}_{\text{Displacement of particles}}-\sum_{i=1}^{N}\frac{\alpha k}{2}\sin(kd_{ij})(\overbrace{a_j^2+a_j^{\dagger 2}+2a_j^\dagger a_j}^{\text{Frequency change of harmonic oscillators}}\underbrace{-a_ja_i-2a_j^\dagger a_i-a_j^\dagger a_i^\dagger}_{\text{Interaction between particles}})\right)+\underbrace{\sum_{i=1}^{N}\hbar\omega_T a_i^\dagger a_i}_{\text{Harmonic oscillator}}$$

- As $\sum_k I(k)\cos(kd)$ is the cosine-part of a fourier series we can generate several symmetric functions by using special intensities and frequencies
- $\sum_k I(k)k\sin(kd)$ is the derivative of this fourier series
- Parts of the Hamiltonian can be tailored by tuning the intensity and the frequency of the transverse pump beam

6 Perturbed eigenvalues and eigenvectors

- Perturbation of the ground state $\bigotimes_{l=1}^{N}|0\rangle^l$ of the harmonic oscillators when transverse pump is treated as a small perturbation

$$|n^1\rangle=\sum_{m\neq n}\frac{\langle m^0|H_1|n^0\rangle}{E_n^0-E_m^0}|m^0\rangle$$

$$=\sum_k\left(\sum_{j=1}^{N}\sum_{i=1}^{j-1}\frac{\epsilon(k)}{\omega_T}\cos\big(kd_{ij}\big)\left(\bigotimes_{l=1,l\neq i}^{N}|0\rangle^l|1\rangle^i-\bigotimes_{l=1,l\neq j}^{N}|0\rangle^l|1\rangle^j\right)+\sum_{j=1}^{N}\sum_{i=1}^{N}\frac{\Omega(k)\sin(kd_{ij})}{4\omega_T}\left(\sqrt{2}\bigotimes_{l=1,l\neq j}^{N}|0\rangle^l|2\rangle^j-\bigotimes_{l=1,l\neq i,j}^{N}|0\rangle^l|1\rangle^i|1\rangle^j\right)\right)$$

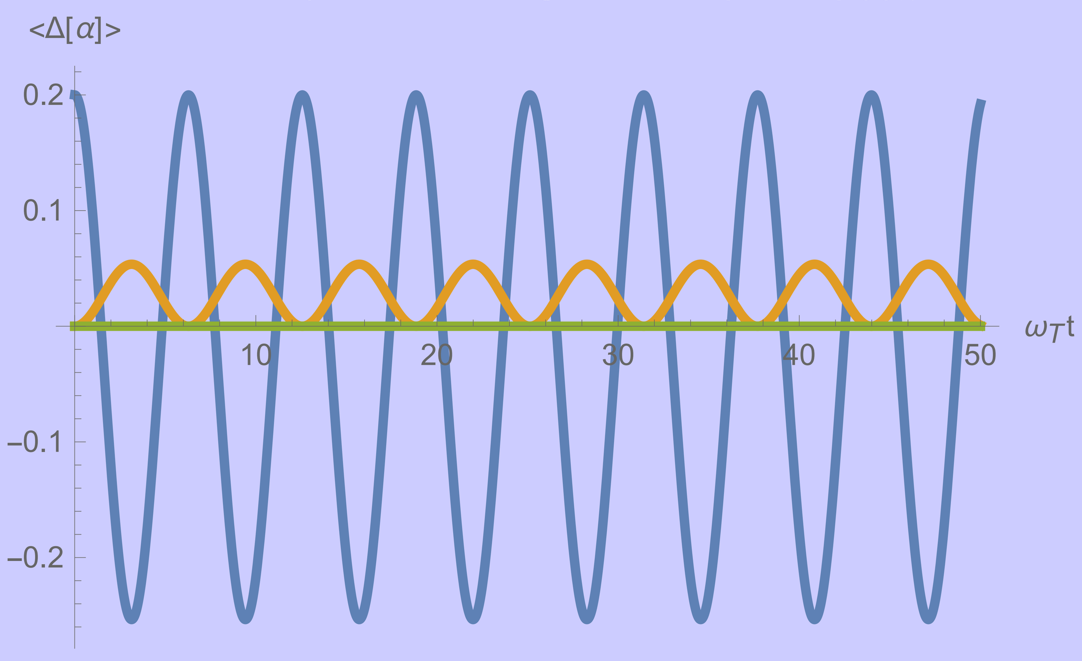
- $\epsilon(k)=\frac{\sigma_{sc}\alpha I(k)}{\hbar c}\approx 6.7\;10^{-3}\omega_T$, $\Omega(k)=\frac{\sigma_{sc}k\alpha^2 I(k)}{\hbar c}\approx 2.2\;10^{-3}\omega_T$
- Perturbation induces states with one and two excitations
- States can be designed by choosing parameters of the transverse pump field (frequency, intensity)
- For example, three particles at distances $x_1=0$, $x_2=0.5\lambda$ and $x_3=1.5\lambda$

$$|n\rangle\propto|000\rangle+\frac{2\epsilon(k)}{\omega_T}(|010\rangle-|100\rangle)$$

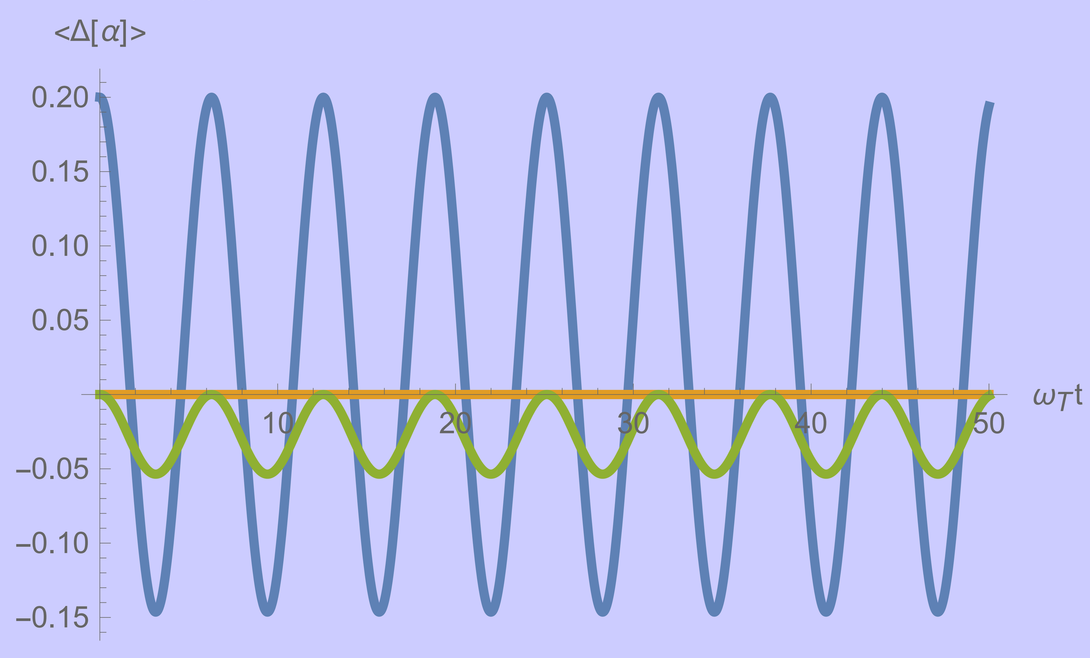
- Suppresses states with two excitations and interactions with the third particle (*)

7 Time evolution

- Time evolution of the displacement of the particles for different frequencies
- First particle in a coherent state, second and third particle in the ground state of the harmonic oscillator $|0\rangle$
- $\langle\Delta_1\rangle$ (blue), $\langle\Delta_2\rangle$ (yellow), $\langle\Delta_3\rangle$ (green) for $|\psi_0\rangle=|\psi_{coh}(\tilde\alpha=0.1),0,0\rangle$, $\epsilon(k)=\frac{\sigma_{sc}\alpha I(k)}{\hbar c}=6.7\cdot 10^{-3}\omega_T$



- $x_1=0$, $x_2=0.5\lambda$ and $x_3=1.5\lambda$
- Coupling between first and second particle as before (*)



- $x_1=0$, $x_2=\lambda$ and $x_3=3\lambda$
- Coupling between first and third particle

- Interaction can be tailored by choosing different frequencies and intensities of the transverse pump beam

8 Outlook

- Design of interaction between particles for quantum simulation
- Use it for quantum information with superpositions of coherent states

9 Acknowledgements and references

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